LESSON 8 – NOTES

- **Ideal Gases Equation of State**
  - $Pv = RT$
  - $PV = mRT$
  - $\frac{PV}{V} = \frac{R}{\mu}$
  - $PV = N\mu T$
  - $m = NM$, $R = \frac{\mu}{M}$, and $M$ is the molecular weight

- **Real Gases – Compressibility Factor**
  - $PV = mZRT$
  - $Pv = ZRT$
  - Law of Corresponding States - $Z = Z\left(\frac{P}{P_c}, \frac{T}{T_c}\right)$
  - Other equations of state are available

**Example:** One-tenth kilogram of air is contained in a spring-loaded piston-cylinder device arranged such that $\frac{\Delta P}{\Delta V} = 10$ MPa/m$^3$. Initially, this air is at 150 kPa, 27 °C. The air now undergoes a process such that the pressure increases to _______ kPa. What is the final temperature of the air?

**System:** One-tenth kilogram of air

**Sketches:**

![Graph showing the process from 150 kPa and 300 K to an unknown pressure and temperature]

**Conditions:** Air is an ideal gas.

**Physical Laws:** $\frac{\Delta P}{\Delta V} = \text{constant}$

$Pv = RT$

**Properties:**

$P_1 = 150$ kPa

$T_1 = 300$ °C

$P_2 = \text{_______}$ kPa

Constant = 10 MPa/m$^3$
Calculations:
From State 1, \( V_1 = \frac{mRT_1}{P_1} = 0.1(8.314/28.97)(300) \) = _______ m\(^3\)

\( \frac{(P_2-P_1)}{(V_2-V_1)} \) = constant. Therefore, \( V_2 = V_1 + \frac{(P_2-P_1)}{constant} \) = _______ + (_______ - 150)/10(1000) = _______ m\(^3\)

For state 2: \( T_2 = \frac{P_2V_2}{mR} = \) _______ (_______) / 0.1(8.314/28.97) = _______ K

• Specific Heats
  o Definition
    \( c_v = \frac{\partial u}{\partial T} \) \text{ and } \( c_p = \frac{\partial h}{\partial T} \)
  o \( \Delta u = \int c_v dT + f(v) \) and \( \Delta h = \int c_p dT + f(P) \)
  o In general, \( c_v = c_v(P,T) \) and \( c_p = c_p(P,T) \)

• Ideal Gas Specific Heats
  o It can be shown that the specific internal energy of an ideal gas depends upon temperature only (i.e. it is not a function of pressure).
  o Then,
    \( c_v = \frac{du}{dT} \) and \( c_p = \frac{dh}{dT} \)

• Ideal Gases with Temperature variable Specific Heats
  \( \Delta u = \int_{T_1}^{T_2} c_v dT, \) Ideal gas tables
  \( \Delta h = \int_{T_1}^{T_2} c_p dT, \) Ideal gas tables

• Ideal Gases with Average and Constant Specific Heats
  o If the specific heats of an ideal gas do not change with temperature or an average value is used,
    \( \Delta u = c_v \Delta T \) and \( \Delta h = c_p \Delta T \)

Example: Two kilograms of 27 °C Nitrogen are contained is a 1.19 m\(^3\) piston-cylinder device arranged to maintain a constant pressure. This nitrogen is now heated until its temperature is ______ °C. How much heat is required for this process?
System: 2 kg of N\(_2\)
Sketches:
Conditions: Quasi-equilibrium, ideal gas, constant c’s

Physical Laws: \( P_v = RT \)

\[ w = \int_1^2 P \, dv \]

\[ q - w = u_2 - u_1 \]

\[ c_p = \frac{dh}{dT}, \quad c_v = \frac{du}{dT} \]

Properties:

\( T_1 = 300 \text{ K} \)

\( T_{12} = \) ________ K

\( M = 2 \text{ kg} \)

\( c_p = 1.039 \text{ kJ/kg-K} \)

\( c_v = 0.743 \text{ kJ/kg-K} \)

\( M = 29 \text{ kg/kg-mol} \)

Calculations:

\( W = P_2v_2 - P_1v_1 \)

\[ q = (u_2 + P_2v_2) - (u_1 + P_1v_1) = h_2 - h_1 = c_p(T_2 - T_1) = 1.039(\_\_\_\_\_\_\_\_\_ - 300) = \_\_\_\_\_\_\_\_\_ \text{ kJ/kg} \]

Then, \( Q = mq = 2 (\_\_\_\_\_\_\_\_\_\_\_) = \_\_\_\_\_\_\_\_\_\_\_ \text{ kJ} \)

- **Ideal Gas Specific Heat – Gas Constant Relations**
  - The enthalpy of an ideal gas is given by
    \[ h = u + P_v = u + RT \]
  - Which when differentiated becomes:
    \[ dh = du + RdT \]
  - Substitution of the ideal gas specific heat relations reduces this to:
    \[ c_p dT = c_v dT + RdT \]
  - Also, \( k \equiv \frac{c_p}{c_v} \)

- **Incompressible Substance Specific Heat**
Since pressure has no effect on the thermodynamic properties of an incompressible substance, we only need one specific heat, \( \frac{du}{dT} \).

\[ \Delta u = \int c dT \]

For constant or average \( c \)'s, \( \Delta u = c_{ave} \Delta T \)

**Incompressible Substance Enthalpy**

- Differentiation of the enthalpy definition yields
  \[ dh = du + vdP + Pdcv \]
  The last term of which is zero since the specific volume of an incompressible substance cannot change.

- Then, \( \Delta h = \int c dT + v \Delta p \)

- For a constant or average specific heat, \( \Delta h = c \Delta T + v \Delta P \)

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**Example:** Tubing and component friction in a hydraulic system (oil, \( c = 1.80 \text{ kJ/kg-K}, v = 0.0011 \text{ m}^3/\text{kg} \)) cause the temperature of the oil to increase from 20 \(^\circ\text{C}\) to _________ \(^\circ\text{C}\) as the pressure increases from 200 \( \text{kPa} \) to 5 \( \text{MPa} \). What is the change in the oil’s specific enthalpy?

**System:** Hydraulic oil

**Sketches:** None Required

**Conditions:** Constant \( c \)'s

**Physical Laws:** \( c_v = du/dT, \Delta h = \Delta u = v \Delta P \)

**Properties:**
- \( c = 1.80 \text{ kJ/kg-K} \)
- \( v = 0.0011 \text{ m}^3/\text{kg} \)
- \( T_1 = 20 ^\circ\text{C}, P_1 = 200 \text{ MPa} \)
- \( T_2 = ______ \text{ ^\circC}, P_2 = 5 \text{ MPa} \)

**Calculations:**
\[
\Delta u = c \Delta T = 1.80(______ - 20) = __________ \text{ kJ/kg}
\]
\[
\Delta h = \Delta u + v \Delta P = __________ + 0.0011(5000 - 200) = __________ \text{ kJ/kg}
\]